Subset sum: the development of a hyperheuristic model in Python.

The problem

Given:

set = [1, 2, 4, 11, 14, 18, 22, 29, 33, 37, 45, 47, 52, 53, 77, 82, 87, 92, 95, 99]

Number of subsets k = 4

Target sum value of subsets = 255 the sum of the set divided by the number of subsets

The subset sum problem is defined as follows: Given a set of integers and a target sum value, determine if there exists a subset of the given set that adds up to the target sum.

A greedy heuristic algorithm

A greedy algorithm is a simple, intuitive algorithm that makes locally optimal choices at each step with the hope of finding a globally optimal solution. It is a popular method for solving optimization

problems, such as the subset sum problem, because it is relatively easy to implement and can often give good results.

Code

```
def greedy_subset_sum(set, k, target_sum):
    # sort the set in descending order
    set = sorted(set, reverse=True)
    subsets = [[] for _ in range(k)]
    # initialize current sum of each subset
    subset_sum = [0 for _ in range(k)]
    # fill the subsets
    for i in range(len(set)):
        # find the subset with the smallest current sum
        min_subset = min(range(k), key=lambda x: subset_sum[x])
        # add the element to the subset
        subsets[min_subset].append(set[i])
        subset_sum[min_subset] += set[i]
    return subsets

# example usage
set = [1, 2, 4, 11, 14, 18, 22, 29, 33, 37, 45, 47, 52, 53, 77, 82, 87, 92, 95, 99]
k = 4
target_sum = 255
print(greedy_subset_sum(set, k, target_sum))
```

This is a simple greedy heuristic algorithm that solves the subset sum problem. The algorithm starts by sorting the set in descending order. Then it initializes k empty subsets and an array to keep track of the current sum of each subset. The algorithm iterates through the sorted set and for each element it finds the subset with the smallest current sum and adds the element to it. The current sum of the subset is then updated. The algorithm returns the subsets as the solution.

It's worth noting that this is a heuristic algorithm, and the solution is not guaranteed to be the optimal solution. The time complexity of this algorithm is O(n*k) where n is the number of elements in the set and k is the number of sub sets. This is because the algorithm iterates through the set once, and for each element it iterates through the subsets once to find the subset with the smallest current sum.

It's also worth noting that this example assumes that the target sum value of the subsets is already known, but in practice it may need to be calculated before running the algorithm. In addition, the example assumes that the subsets must be disjoint, but in some cases overlapping subsets may be allowed.

In conclusion, this greedy heuristic algorithm is a simple approach to solving the subset sum problem by iterating through the elements of a set and adding them to the subset with the smallest current sum. However, the solution may not always be optimal and the time complexity of the algorithm is O(n*k).

Refined greedy algorithm with error checking

```
def greedy_subset_sum(set, k):
```

```
    if not set:
        print("Error: set is empty")
        return
    if sum(set) % k != 0:
        print("Error: sum of set is not divisible by number of subsets")
        return
    # calculate the target sum value of subsets
    target_sum = sum(set) // k
    # sort the set in descending order
    set = sorted(set, reverse=True)
    subsets = [[] for _ in range(k)]
    # initialize current sum of each subset
    subset_sum = [0 for _ in range(k)]
    # fill the subsets
    for i in range(len(set)):
        # find the subset with the smallest current sum
        min_subset = min(range(k), key=lambda x: subset_sum[x])
        # add the element to the subset
        subsets[min_subset].append(set[i])
        subset_sum[min_subset] += set[i]
    return subsets

# example usage
set = [1, 2, 4, 11, 14, 18, 22, 29, 33, 37, 45, 47, 52, 53, 77, 82, 87, 92, 95, 99]
k = 4
print(greedy_subset_sum(set, k))
```

This updated version of the algorithm first checks if the input set is empty and if not, it checks if the sum of the set is divisible by the number of subsets k. If either of these checks fail, the algorithm returns an error message and stops. If both checks pass, it calculates the target sum value of subsets by dividing the sum of the set by the number of subsets. The rest of the algorithm is the same as the previous version, it sorts the set in descending order, initializes k empty subsets and an array to keep track of the current sum of each subset. The algorithm iterates through the sorted set and for each element it finds the subset with the smallest current sum and adds the element to it. The current sum of the subset is then updated. The algorithm returns the subsets as the solution.

This version of the algorithm includes more checks for input validation, and also calculates the target sum value of subsets, which allows for more flexibility in the input set and number of subsets. The time complexity of this updated version of the algorithm is also O(n*k) where n is the number of elements in the set and k is the number of subsets.

The best fit development

The best fit algorithm is a meta-heuristic algorithm that aims to make the subsets as balanced as possible by adding the element that makes the current sum of the subset closest to the target sum. The algorithm starts by calculating the target sum value of subsets by dividing the sum of the set by the number of subsets. Then it sorts the set in descending order, initializes k empty subsets and an array to keep track of the current sum of each subset. The algorithm iterates through the sorted set and for each element it finds the subset with the closest current sum to the target sum and adds the

element to it. The current sum of the subset is then updated. The algorithm returns the subsets as the solution.

In the example provided, the best fit algorithm was used to solve the subset sum problem by dividing the given set of numbers into k subsets with equal sum values. The algorithm starts by calculating the target sum value and then it sorts the set in descending order. Then it initializes k empty subsets and an array to keep track of the current sum of each subset. It then iterates through the sorted set, for each element it finds the subset with the closest current sum to the target sum and adds the element to it. The current sum of the subset is then updated. The algorithm returns the subsets as the solution.

This version of the algorithm is more efficient as compared to the greedy algorithm because it tries to balance the subsets as much as possible, which results in a more balanced solution. The time complexity of this version of the algorithm is also O(n*k) where n is the number of elements in the set and k is the number of subsets.

Code

```
import random

def best_fit_subset_sum(set, k):
    if not set:
        print("Error: set is empty")
        return
    if sum(set) % k != 0:
        print("Error: sum of set is not divisible by number of subsets")
        return
    # calculate the target sum value of subsets
    target_sum = sum(set) // k
    # sort the set in descending order
    set = sorted(set, reverse=True)
    subsets = [[] for _ in range(k)]
    # initialize current sum of each subset
    subset_sum = [0 for _ in range(k)]
    # fill the subsets using best fit heuristic
    for i in range(len(set)):
        # find the subset with the closest current sum to the target sum
        closest_subset = min(range(k), key=lambda x: abs(subset_sum[x] - target_sum))
        # add the element to the subset
        subsets[closest_subset].append(set[i])
        subset_sum[closest_subset] += set[i]
    return subsets

# example usage
set = [1, 2, 4, 11, 14, 18, 22, 29, 33, 37, 45, 47, 52, 53, 77, 82, 87, 92, 95, 99]
k = 4
print(best_fit_subset_sum(set, k))
```

This version of the algorithm uses the best fit heuristic which is a meta-heuristic algorithm that aims to make the subsets as balanced as possible by adding the element that makes the current sum of

the subset closest to the target sum. The best fit heuristic is a simple and efficient algorithm that can improve the performance of the solution as compared to the greedy heuristic algorithm because it takes into account the target sum value and tries to balance the subsets as much as possible.

It starts by checking if the input set is empty, and if not it checks if the sum of the set is divisible by the number of subsets k. If either of these checks fail, the algorithm returns an error message and stops. If both checks pass, it calculates the target sum value of subsets by dividing the sum of the set by the number of subsets. Then it sorts the set in descending order, initializes k empty subsets and an array to keep track of the current sum of each subset. The algorithm iterates through the sorted set and for each element it finds the subset with the closest current sum to the target sum and adds the element to it. The current sum of the subset is then updated. The algorithm returns the subsets as the solution.

This version of the algorithm is more efficient as compared to the greedy algorithm because it tries to balance the subsets as much as possible, which results in a more balanced solution. The time complexity of this version of the algorithm is also O(n*k) where n is the number of elements in the set and k is the number of subsets.

The development of a hyperheuristic

A best fit hyperheuristic algorithm is a higher-level strategy that selects the best low-level heuristic for a given problem instance. In the example provided, the best fit hyperheuristic algorithm was used to improve the performance of the subset sum problem by selecting the best performing heuristic between the greedy and best fit meta-heuristic algorithms. The algorithm starts by trying both greedy and best fit heuristics, and then calculates the deviation (i.e. difference) between the current sum of the subsets and the target sum for each heuristic. The algorithm then selects the heuristic with the lowest deviation as the best performing heuristic and returns the subsets found by this heuristic.

This example illustrates how a hyperheuristic algorithm can be implemented, it uses a combination of different heuristics, such as the greedy and best fit heuristics, and then evaluate their performance on different problem instances. The hyperheuristic algorithm then selects the best performing heuristic for each problem instance. This helps in providing better performance as compared to the greedy and best fit heuristics alone. The time complexity of this hyper-heuristic is also O(n*k) where n is the number of elements in the set and k is the number of subsets.

Code

```
def best_fit_hyperheuristic(set, k):
    if not set:
        print("Error: set is empty")
        return
    if sum(set) % k != 0:
        print("Error: sum of set is not divisible by number of subsets")
        return
    target_sum = sum(set) // k
    # Try both greedy and best fit heuristics
    greedy_subsets = greedy_subset_sum(set, k)
    greedy_deviation = sum([abs(target_sum - sum(subset)) for subset in greedy_subsets])
```

```
    best_fit_subsets = best_fit_subset_sum(set, k)
    best_fit_deviation = sum([abs(target_sum - sum(subset)) for subset in best_fit_subsets])
    # Select the best performing heuristic
    if greedy_deviation < best_fit_deviation:
        return greedy_subsets
    else:
        return best_fit_subsets

# example usage
set = [1, 2, 4, 11, 14, 18, 22, 29, 33, 37, 45, 47, 52, 53, 77, 82, 87, 92, 95, 99]
k = 4
print(best_fit_hyperheuristic(set, k))
```

This example uses the greedy and best fit heuristics to find the subsets and then calculates the deviation (i.e. difference) between the current sum of the subsets and the target sum for each heuristic. The hyperheuristic algorithm selects the heuristic with the lowest deviation as the best performing heuristic. Finally, it returns the subsets found by the best performing heuristic.

It's worth noting that this example is a simple illustration of how a hyperheuristic algorithm can be implemented, and that different problem instances may require different heuristics, or a different combination of heuristics, to achieve the best performance.

This example uses the best fit hyperheuristic algorithm which is a higher-level strategy that selects the best low-level heuristic for a given problem instance. This helps in providing better performance as compared to the greedy and best fit heuristics alone. The time complexity of this hyper-heuristic is also O(n*k) where n is the number of elements in the set and k is the number of subsets.

Complete code

```
import random

def greedy_subset_sum(set, k):
    if not set:
        print("Error: set is empty")
        return
    if sum(set) % k != 0:
        print("Error: sum of set is not divisible by number of subsets")
        return
    # calculate the target sum value of subsets
    target_sum = sum(set) // k
    # sort the set in descending order
    set = sorted(set, reverse=True)
    subsets = [[] for _ in range(k)]
    # initialize current sum of each subset
    subset_sum = [0 for _ in range(k)]
    # fill the subsets
    for i in range(len(set)):
        # find the subset with the smallest current sum
        min_subset = min(range(k), key=lambda x: subset_sum[x])
        # add the element to the subset
        subsets[min_subset].append(set[i])
```

```
        subset_sum[min_subset] += set[i]
    return subsets

def best_fit_subset_sum(set, k):
    if not set:
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    if sum(set) % k != 0:
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    # calculate the target sum value of subsets
    target_sum = sum(set) // k
    # sort the set in descending order
    set = sorted(set, reverse=True)
    subsets = [[] for _ in range(k)]
    # initialize current sum of each subset
    subset_sum = [0 for _ in range(k)]
    # fill the subsets using best fit heuristic
    for i in range(len(set)):
        # find the subset with the closest current sum to the target sum
        closest_subset = min(range(k), key=lambda x: abs(subset_sum[x] - target_sum))
        # add the element to the subset
        subsets[closest_subset].append(set[i])
        subset_sum[closest_subset] += set[i]
    return subsets

def best_fit_hyperheuristic(set, k):
    if not set:
        print("Error: set is empty")
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        print("Error: sum of set is not divisible by number of subsets")
        return
    target_sum = sum(set) // k
    # Try both greedy and best fit heuristics
    greedy_subsets = greedy_subset_sum(set, k)
    greedy_deviation = sum([abs(target_sum - sum(subset)) for subset in greedy_subsets])
    best_fit_subsets = best_fit_subset_sum(set, k)
    best_fit_deviation = sum([abs(target_sum - sum(subset)) for subset in best_fit_subsets])
    # Select the best performing heuristic
    if greedy_deviation < best_fit_deviation:
        return greedy_subsets
    else:
        return best_fit_subsets

# example usage
set = [1, 2, 4, 11, 14, 18, 22, 29, 33, 37, 45, 47, 52, 53, 77, 82, 87, 92, 95, 99]
k = 4
print(best_fit_hyperheuristic(set, k))
```

Time and Space Complexities

The time complexity of a greedy algorithm for solving the subset sum problem is typically O(n*2^n), where n is the number of elements in the set. This is because the algorithm needs to check all possible subsets of the set in order to find a solution.

The time complexity of the best fit meta-heuristic algorithm for solving the subset sum problem is typically O(n*k), where n is the number of elements in the set and k is the number of subsets. The algorithm iterates through the sorted set, for each element it finds the subset with the closest current sum to the target sum and adds the element to it. The current sum of the subset is then updated.

The time complexity of the best fit hyperheuristic algorithm for solving the subset sum problem is also typically O(n*k), where n is the number of elements in the set and k is the number of subsets. The algorithm starts by trying both greedy and best fit heuristics, and then calculates the deviation (i.e. difference) between the current sum of the subsets and the target sum for each heuristic. The algorithm then selects the heuristic with the lowest deviation as the best performing heuristic and returns the subsets found by this heuristic.

In terms of space complexity, both the greedy and best fit meta-heuristic algorithms for solving the subset sum problem have a space complexity of O(nk) as the algorithm needs to store the subsets and current sum of each subset. The best fit hyperheuristic algorithm also has a space complexity of O(nk) as it needs to store the subsets and current sum of each subset, as well as the deviation of each heuristic.

Conclusion

Today, we have discussed several different algorithms that can be used to solve the subset sum problem, including greedy heuristics, best fit meta-heuristics, and best fit hyperheuristics.

A greedy algorithm is a simple, intuitive algorithm that makes locally optimal choices at each step with the hope of finding a globally optimal solution. It is a popular method for solving optimization problems because it is relatively easy to implement and can often give good results. In the example provided, we have seen a python implementation of the greedy algorithm for solving the subset sum problem.

A best fit meta-heuristic algorithm is an algorithm that aims to make the subsets as balanced as possible by adding the element that makes the current sum of the subset closest to the target sum. In the example provided, we have seen a python implementation of the best fit algorithm for solving the subset sum problem.

A best fit hyperheuristic algorithm is a higher-level strategy that selects the best low-level heuristic for a given problem instance. In the example provided, we have seen a python implementation of the best fit hyperheuristic algorithm which is used to improve the performance of the subset sum problem by selecting the best performing heuristic between the greedy and best fit meta-heuristic algorithms.

In general, the use of a hyperheuristic can be beneficial when dealing with complex and dynamic problems, as it can adapt to the specific characteristics of the problem at hand, and can be more efficient than the other algorithms alone.

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